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SCALING OF A BLAST FROM ONE MEDIUM TO ANOTHER

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ABSTRACT

The problem of finding the most general laws by means of which a blast in one medium can be scaled to a "similar" blast in another medium is here investigated. It is found that the following conditions are sufficient; they also appear to be necessary.

1.) Both media must have the same equation of state and this equation must, in general, be of the form $E_{int} = \frac{pV}{\gamma - 1}$. If the initial pV products for both media are the same, then the more general equation of state $E = pVf(pV)$ is admissible.

2.) The initial pressures in the two media must be space independent.

3.) The initial densities must be space independent.

The actual analysis is carried out for a simplified case, but will obviously extend to the above generalization and no farther.

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We can write the hydrodynamical equations in terms of $\xi = P/p_o$, $\eta = \rho/\rho_o$ and the scaled time $\tau = t\sqrt{p_o V_o}$. In spherical coordinates they are

$$1 = \eta\left(\frac{R}{r}\right)^2 \frac{\partial R}{\partial r}$$
$$\frac{\partial^2 R}{\partial \tau^2} = -\left(\frac{R}{r}\right)^2 \frac{\partial \xi}{\partial r} \qquad 1$$
$$\frac{\partial \xi}{\partial \tau} - \frac{\gamma}{\eta}\frac{\partial \eta}{\partial \tau} = 0$$

We have assumed that the internal energy is $pV/(\gamma - 1)$. The Hugoniot conditions become

$$\frac{dY}{d\tau} = \sqrt{\eta_s \cdot \frac{\xi_s - 1}{\eta_s - 1}} \qquad 2$$
$$\eta_s = \frac{(\gamma + 1)\xi_s + (\gamma - 1)}{(\gamma - 1)\xi_s + (\gamma + 1)}$$

where the subscript s refers to conditions at the shock front, and Y is shock radius. We see then that the differential equations (1) and the boundary conditions (2) are independent of the initial conditions of the medium if we agree to measure all velocities in the τ-time scale.

Consider now two explosions, occurring in media of different initial densities and pressures, which are similar at some instant in the following sense:

a.) Both shock fronts are at identical radii

b.) Both blast waves have identical contours of ξ and η at this instant of time.

Both blasts then satisfy identical initial conditions and identical boundary conditions; thus both will develop (in τ-time) in identical fashions. Thus we see that if both are similar at $t = 0$, both will be similar at respective later times t_1 , t_2 if

$$t_2 = \sqrt{\frac{p_1 V_1}{p_2 V_2}}\ t_1 .$$

The subscripts on p, V refer to the initial conditions of the two media.

The scale factors for the various variables between two similar explosions have for the most part been found; for ξ , η , and R they are unity, while for t the scale factor is $\sqrt{pV}$, which is sound velocity for a gamma law medium. It remains to find the ratio between the total energies in two similar explosions. The internal energy of a unit mass is

$$P\,E = \int_{p_o, V_o}^{p, V} p\,dV = -\,p_o V_o \int_{1,\,1}^{\xi,\,\eta} \frac{\xi}{\eta^2}\, d\eta$$

and the kinetic energy per unit mass is

$$KE = \tfrac{1}{2}\, U^2 = \tfrac{1}{2}\, p_o V_o \left(\frac{d\,R}{d\tau}\right)^2$$

The total energy per unit mass of any particular parcel is then

$$E_{tot} = p_o V_o \cdot A$$

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where A is a function only of the scale invariant variables ξ, η, and R. Consider now two explosions, similar at the instant under consideration. Explosion (1) takes place in a medium initially under the conditions (p_1, V_1); the initial conditions for explosion (2) are (p_2, V_2). For the total energies we have

$$W_1 = \rho_1 \cdot p_1 V_1 \int_0^Y A . 2\pi R^2 \, dR$$

$$W_2 = \rho_2 \cdot p_2 V_2 \int_0^Y A . 2\pi R^2 \, dR$$

or

$$\frac{W_1}{P_1} = \frac{W_2}{P_2} \qquad 3$$

This last equation gives the relationship between the energy releases required of two explosions in order that they may be similar.

We see now how to scale from an explosion in a medium of one density to an explosion in a similar medium at another density. Suppose, for example, we wish to obtain a pressure vs. distance curve for a 50 kiloton explosion occurring at an altitude of 9 kilometers. The pressure at 9 kilometers is 0.312 atmospheres. According to equation (3), an explosion of $\frac{50}{.312} = 160$ kilotons at

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sea level will be similar to 50 kilotons at 9 kilometers. By the usual $W^{1/3}$ law of scaling we construct a pressure vs. distance curve for a 160 kiloton yield. Say that the result is $p_s(R)$. The result for a 50 kiloton yield at 9 kilometers will be $0.312 \cdot p_s(R)$, (equal values of ξ_{shock} are attained at equal shock radii). In order to show the effect we have plotted in Fig. I, the over pressure vs. distance curves for a 50 kiloton yield at sea level and at 9 kilometers altitude. Both curves are for free air, i. e., they each assume an infinite homogeneous atmosphere. It is readily seen that significant over-pressures are achieved at larger distances in a denser medium.

The foregoing laws for scaling explosions from one medium to another have assumed

a.) The flow is spherically symmetrical.

b.) Both media obey the gamma law and have the same value of gamma.

c.) Both media are infinite and homogeneous.

Restriction (a) is obviously unnecessarily stringent. It is immediately apparent that no assumptions need be made concerning symmetry; the laws hold for any type of flow. Condition (b) is necessary in general, in order that the shock conditions (2) be independent of the initial

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conditions. In the special case that the initial pV products for the two media are the same, gamma maybe an arbitrary function of the product pV. Condition (c) may not be relaxed, as the derivation of the equations of motion (1) assume communtability of the derivative operators with p_o and V_o.

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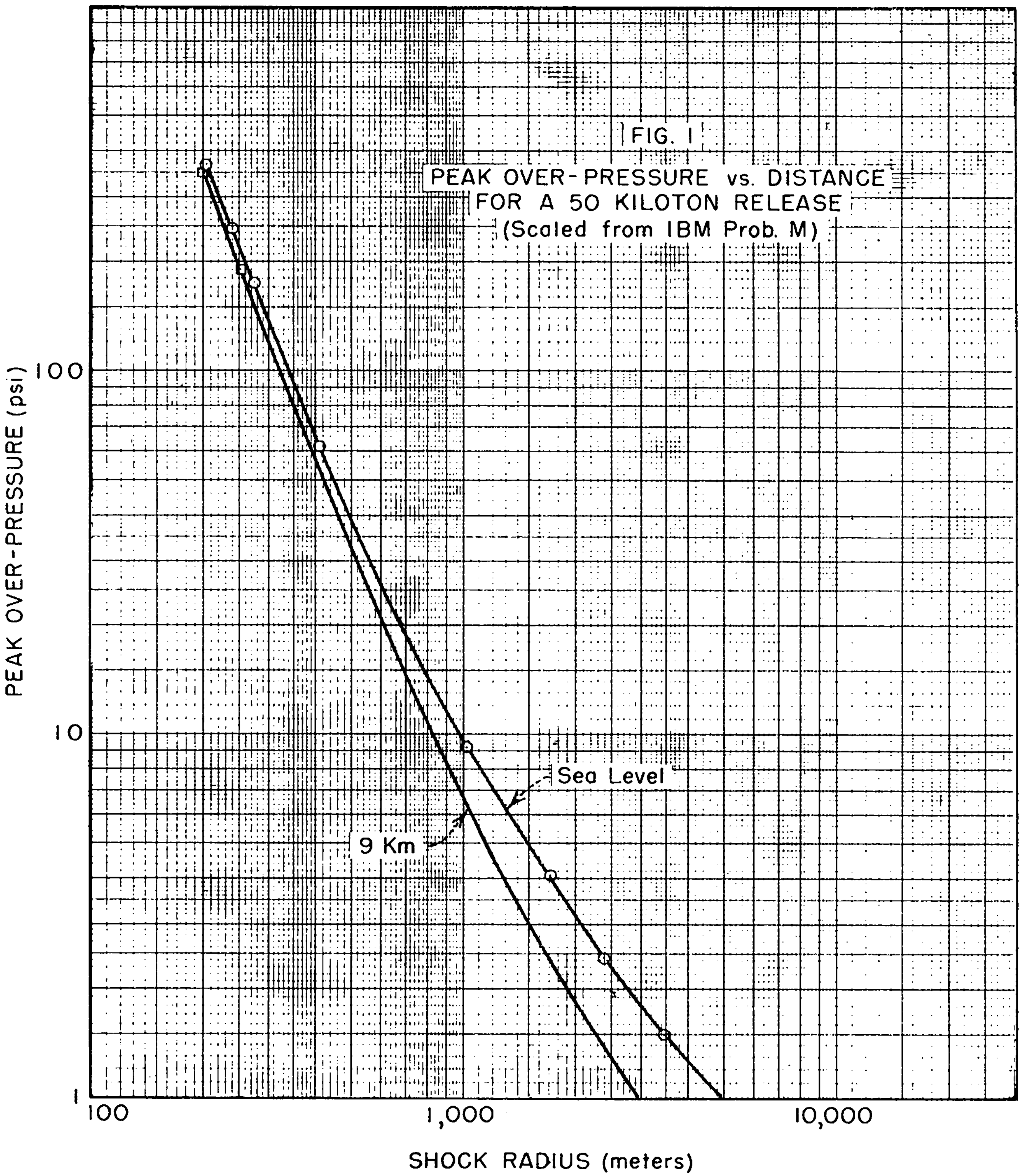

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